HOMEWORK 4

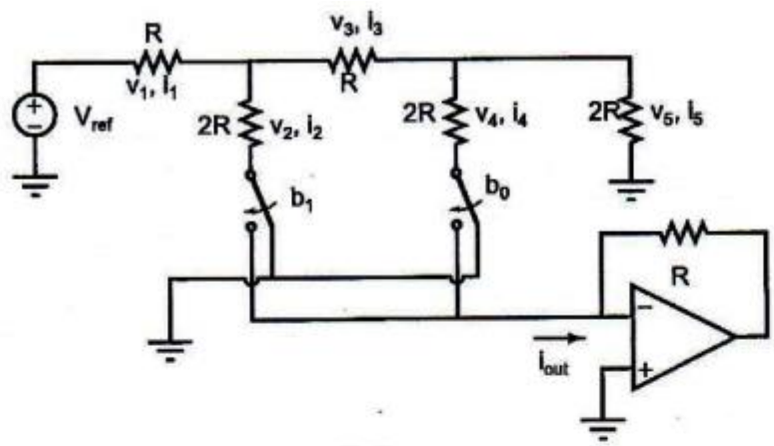
ECE 580

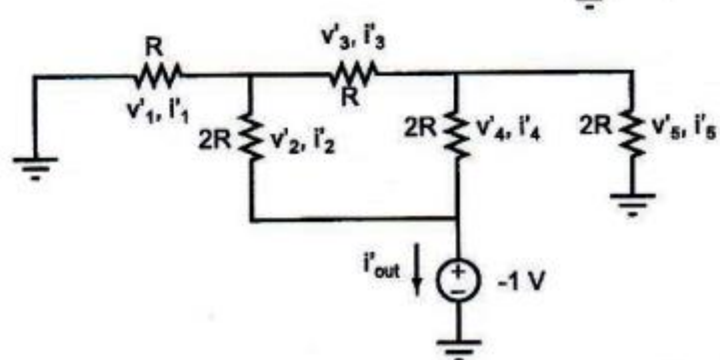
Due November 8, 2023

The circuit shown on top is a two-bit digital-to-analog converter. V_{ref} = 3 V, and R = 1 kΩ. The opamp is assumed to be ideal. The effects of resistance errors in the ladder need to be found, using the adjoint network shown at the bottom.

1. Analyze both circuits.
2. Find the sensitivities to errors in the ladder resistors.

Hint: use the MNA to find the currents in the adjoint network.





Solution:

For circuit (A), MNA can be applied to get the node voltages.

$$\begin{bmatrix} \frac{1}{R} & -\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{2R}\right) & \frac{1}{R} & \frac{1}{2R} \\ 0 & \frac{1}{R} & -(\frac{1}{R}+\frac{1}{2R}+\frac{1}{2R}) & \frac{1}{2R} \\ 0 & \frac{1}{2R} & \frac{1}{2R} & -(\frac{1}{2R}+\frac{1}{2R}) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_{out} \end{bmatrix}$$

where $v_a = V_{ref}, v_d = 0$.

Alternatively, notice that the circuit has a R-2R DAC, the node voltage can be easily obtained as

$$v_a = V_{ref}, v_b = \frac{1}{2}V_{ref}, v_c = \frac{1}{4}V_{ref}, v_d = 0$$

Therefore,

$$v_1 = \frac{1}{2}V_{ref}, v_2 = \frac{1}{2}V_{ref}, v_3 = \frac{1}{4}V_{ref}, v_4 = \frac{1}{4}V_{ref}, v_5 = \frac{1}{4}V_{ref}$$

$$i_1 = \frac{V_{ref}}{2R}, i_2 = \frac{V_{ref}}{4R}, i_3 = \frac{V_{ref}}{4R}, i_4 = \frac{V_{ref}}{8R}, i_5 = \frac{V_{ref}}{8R}$$

And,

$$i_{out} = \frac{v_b}{2R} + \frac{v_c}{2R} = \frac{3V_{ref}}{8R}$$

$$v_O = -i_{out}R = -\frac{3V_{ref}}{8}$$

For circuit (B), MNA

$$\begin{bmatrix} \frac{1}{R} & -\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{2R}\right) & \frac{1}{R} & \frac{1}{2R} \\ 0 & \frac{1}{R} & -(\frac{1}{R}+\frac{1}{2R}+\frac{1}{2R}) & \frac{1}{2R} \\ 0 & \frac{1}{2R} & \frac{1}{2R} & -(\frac{1}{2R}+\frac{1}{2R}) \end{bmatrix} \begin{bmatrix} v_a' \\ v_b' \\ v_c' \\ v_d' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_{out}' \end{bmatrix}$$

where $v_a' = 0, v_d' = -1V$.

The node voltage can be solved as

$$v_a' = 0, v_b' = -\frac{3}{8}V\ , v_c' = -\frac{7}{16}V, v_d' = -1V$$

Alternatively, due to the property of R-2R DAC,

$$v_b' = \frac{1}{4}v_d' + \frac{1}{8}v_d' = -\frac{3}{8}V$$

Solve node c’ voltage by applying voltage divider,

$$v_c' = \frac{\frac{v_b'}{R} + \frac{v_d'}{2R} + \frac{0}{2R}}{\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}} = -\frac{7}{16}V$$

Therefore,

$$v_1' = \frac{3}{8}V, v_2' = \frac{5}{8}V, v_3' = \frac{1}{16}V, v_4' = \frac{9}{16}V_{ref}, v_5' = -\frac{7}{16}V$$

$$i_1' = \frac{3}{8R}, i_2' = \frac{5}{16R}, i_3' = \frac{1}{16R}, i_4' = \frac{9}{32R}, i_5' = -\frac{7}{32R}$$

Applying Tellegen’s Theorem, we have

$$\Delta\boldsymbol{v}^T\boldsymbol{i}' - \boldsymbol{v}'^T\Delta\boldsymbol{i} = 0$$

i.e.,

$$(i_1\Delta R i_1' + i_2\Delta 2R i_2' + i_3\Delta R i_3' + i_4\Delta 2R i_4' + i_5\Delta 2R i_5') - (-1V)\Delta i_{out} = 0$$

Therefore, the sensitivities of the output current (i_{out}) to the resistances are

$$\Delta i_{out} = -\frac{V_{ref}}{2R}\frac{3}{8R}\Delta R - \frac{V_{ref}}{4R}\frac{5}{16R}\Delta 2R - \frac{V_{ref}}{4R}\frac{1}{16R}\Delta R - \frac{V_{ref}}{8R}\frac{9}{32R}\Delta 2R + \frac{V_{ref}}{8R}\frac{7}{32R}\Delta 2R$$

$$= \left(-\frac{9}{16}\Delta R - \frac{15}{64}\Delta 2R - \frac{3}{64}\Delta R - \frac{27}{256}\Delta 2R + \frac{21}{256}\Delta 2R\right)\ A\cdot M\Omega^{-1}$$